

## LA-UR-14-21608

Approved for public release; distribution is unlimited.

Title: Adjoint Weighting Methods Applied to Monte Carlo Simulations of Applications and Experiments in Nuclear Criticality

Author(s): Kiedrowski, Brian C.

Intended for: Univ. of Michigan Seminar

Issued: 2014-03-11



**Disclaimer:**

Los Alamos National Laboratory, an affirmative action/equal opportunity employer, is operated by the Los Alamos National Security, LLC for the National Nuclear Security Administration of the U.S. Department of Energy under contract DE-AC52-06NA25396. By approving this article, the publisher recognizes that the U.S. Government retains nonexclusive, royalty-free license to publish or reproduce the published form of this contribution, or to allow others to do so, for U.S. Government purposes. Los Alamos National Laboratory requests that the publisher identify this article as work performed under the auspices of the U.S. Department of Energy. Los Alamos National Laboratory strongly supports academic freedom and a researcher's right to publish; as an institution, however, the Laboratory does not endorse the viewpoint of a publication or guarantee its technical correctness.

# Adjoint Weighting Methods Applied to Monte Carlo Simulations of Applications and Experiments in Nuclear Criticality

Brian C. Kiedrowski

Los Alamos National Laboratory

March 13, 2014

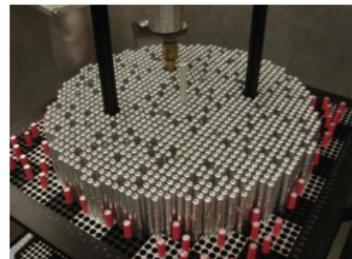
# Outline

- 1 Introduction & Basics
- 2 Adjoint Weighting in Continuous-Energy Monte Carlo
- 3 Point Kinetics
- 4 Sensitivity/Uncertainty
- 5 Graduate Student Research
- 6 Future Research

## My Goals of Methods Development

- Develop Monte Carlo radiation transport methods and simulation software for engineering analysis that are robust, efficient, and easy to use.
- Provide computational resources to assess and improve the predictive capability of radiation transport methods and nuclear data.
- Focus of my research is on criticality and applications that use fissionable material.

# Applications Involving Fissionable Material



## Nuclear Criticality – Why it Matters

- When nuclear criticality is achieved, a system is in a configuration such that a runaway nuclear chain reaction will occur.
- Resulting radiation doses are often lethal and cannot be mitigated by time, distance, and shielding.
- Small changes in configuration can be the difference between life and death.
- We have no innate sensing ability to detect how close we are to criticality.
- **Handling fissionable material without criticality controls is like walking near a cliff on a very foggy day.**

## How to Predict Criticality

- Experiments
  - Ideal option, and only one in the early days.
  - Difficult and expensive to perform in modern regulatory environment.
  - Few facilities available today, but large databases are available for criticality.
- Simulation
  - Not as good as reality, but with modern computing, this has become the preferred option.
  - Mature software, nuclear data, computational platforms.
  - Often, **but not always**, good quantitative agreement with experiment.
- Experiments and simulation supplement each other.

## Assessing Criticality with Simulations

- Solve  $k$ -eigenvalue form of the linear radiation transport equation (RTE):

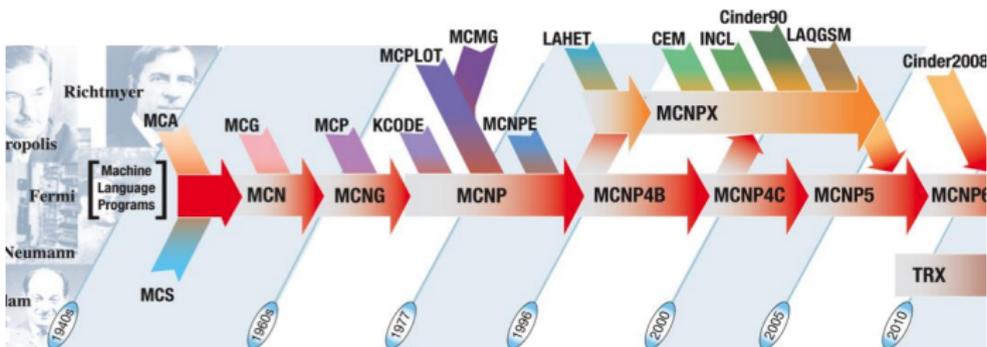
$$\begin{aligned} (\hat{\Omega} \cdot \nabla + \Sigma_t) \psi(\mathbf{r}, \hat{\Omega}, E) = & \iint dE' d\Omega' \Sigma_s(E' \rightarrow E, \hat{\Omega}' \cdot \hat{\Omega}) \psi(\mathbf{r}, \hat{\Omega}', E') \\ & + \frac{1}{k_{\text{eff}}} \iint dE' d\Omega' \chi(E' \rightarrow E) \nu \Sigma_f(E') \psi(\mathbf{r}, \hat{\Omega}', E') \end{aligned}$$

- Find fundamental eigenvalue  $k_{\text{eff}}$  and corresponding eigenfunction.
- $k_{\text{eff}}$  is a mathematical factor that balances gains and losses.
- $\psi$  represents the neutron distribution for this adjusted system.
- Two classes of methods available for solving this equation:
  - Deterministic (diffusion theory,  $P_n$ ,  $S_n$ , MOC, etc.)
  - Monte Carlo

## Monte Carlo Basics

- The Monte Carlo method does not solve the RTE in a direct, mathematical sense.
- Rather, it simulates the underlying radiation physics that the equation describes.
  - The neutron transport equation describes mean-value radiation behavior.
  - Monte Carlo simulation makes counts of responses (e.g., reaction rates) from the radiation physics and takes the mean average.
  - Because these correspond, the solution to the transport equation can be inferred from the Monte Carlo simulation.
- The eigenvalue problem is solved iteratively
  - Guess  $k_{\text{eff}}$  and  $\psi$ , transport neutrons one fission generation (an iteration), estimate  $k_{\text{eff}}$  and store fission neutrons as the new source.
  - After many iterations, the process converges and results may be obtained.

# MCNP



- General-purpose, production Monte Carlo radiation transport software developed and maintained by Los Alamos National Laboratory.
- Continuous-energy physics supporting both neutral and charged particle transport.
- Wide range of applications: criticality safety, shielding, research reactors, medical physics, high-energy physics, stockpile stewardship, etc.
- Available from RISCC. Over 10,000 users.

# Outline

- 1 Introduction & Basics
- 2 Adjoint Weighting in Continuous-Energy Monte Carlo**
- 3 Point Kinetics
- 4 Sensitivity/Uncertainty
- 5 Graduate Student Research
- 6 Future Research

## Adjoint of the RTE

- Adjoint RTE reverses direction of streaming, scattering, and fission:

$$\begin{aligned} & \left( -\hat{\Omega} \cdot \nabla + \Sigma_t \right) \psi^\dagger(\mathbf{r}, \hat{\Omega}, E) \\ & - \iint dE' d\Omega' \Sigma_s(E \rightarrow E', \hat{\Omega} \cdot \hat{\Omega}') \psi^\dagger(\mathbf{r}, \hat{\Omega}', E') \\ & = \frac{\nu \Sigma_f(E)}{k_{\text{eff}}} \iint dE' d\Omega' \chi(E \rightarrow E') \psi^\dagger(\mathbf{r}, \hat{\Omega}', E') \end{aligned}$$

- Adjoint function  $\psi^\dagger$  can be thought of as the importance with respect to the right-hand side of the equation, the fission source.

## $k$ -Eigenvalue Importance: Thought Experiment

- The importance is the propensity of neutrons at  $\mathbf{r}, \hat{\Omega}, E$  toward driving the self-sustaining chain reaction.
- Consider the following thought experiment:
  - A critical assembly starts with no neutrons.
  - Insert a neutron into the assembly at  $\mathbf{r}, \hat{\Omega}, E$  and wait a “long time”.
  - Take a count of the number of neutrons in the assembly and record this number.
  - Flush the neutrons out of the system, and repeat this over and over.
  - The mean of the counts is proportional to the importance at  $\mathbf{r}, \hat{\Omega}, E$ .
- What if the system is not critical?
  - System considered is a mathematical model where  $1/k_{\text{eff}}$  balances losses and gains.

## Why Care About the Adjoint Function in Criticality?

- Practically speaking, the adjoint function has convenient mathematical properties that allow the construction of simple models to estimate:
  - Kinetic behavior of reactors near criticality.
  - Perturbations for small changes in reactor configuration.
  - Sensitivity coefficients for uncertainty quantification.
- These models have terms that are adjoint-weighted integrals over some domain:

$$\langle \psi^\dagger, A\psi \rangle$$

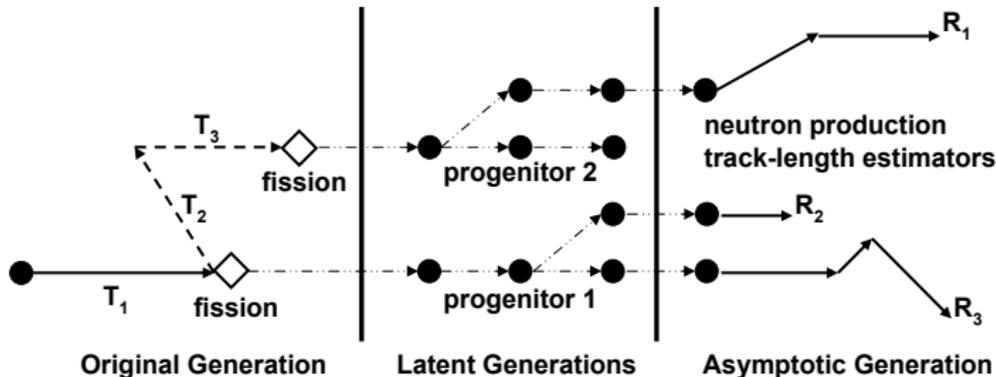
- Goal is to develop a method that can evaluate these integrals in a forward simulation with minimal additional computational cost.

## Adjoint Weighting During Forward MC Simulation

- Eigenvalue calculations are an iterative process.
- Any state during a Monte Carlo random walk can be thought of as a point where a neutron is introduced into the system.
- If these states can be tracked and linked to their future behavior after several generations, estimates of the importance of those events can be made.
- User must decide how many generations constitute a “long time”.
  - Empirical studies show 5-10 generations sufficient for most problems.
- Otherwise, no mesh or discretization is needed beyond that already in the forward simulation.

## Adjoint Weighting During Forward MC Simulation

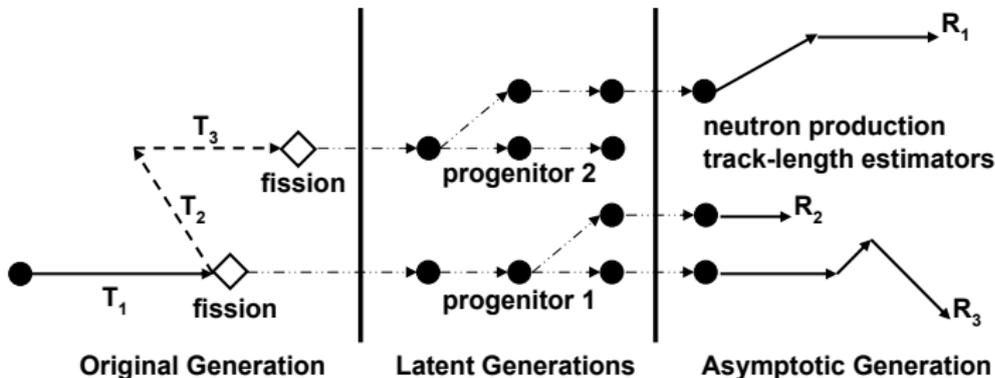
- In the first (original) iteration (generation):
  - Record events representing the kernel times the neutron flux (normal Monte Carlo tallies).
  - For each recorded event, tag the neutrons, and associate tags with their events.
  - Events and tags should be combined as possible to limit storage requirements.
  - Progeny of neutrons from fission inherit their tags.





## Adjoint Weighting During Forward MC Simulation

- In the final (asymptotic) iteration in the block:
  - Neutron populations are assumed to be converged to their asymptotic value (i.e., infinite time).
  - Record the number of neutrons produced by a track-length estimator of fission neutron production.
  - Multiply these by their appropriate scores from recorded events to estimate the importance-weighted integrals by Monte Carlo.
  - Repeat the process until desired statistical precision obtained.



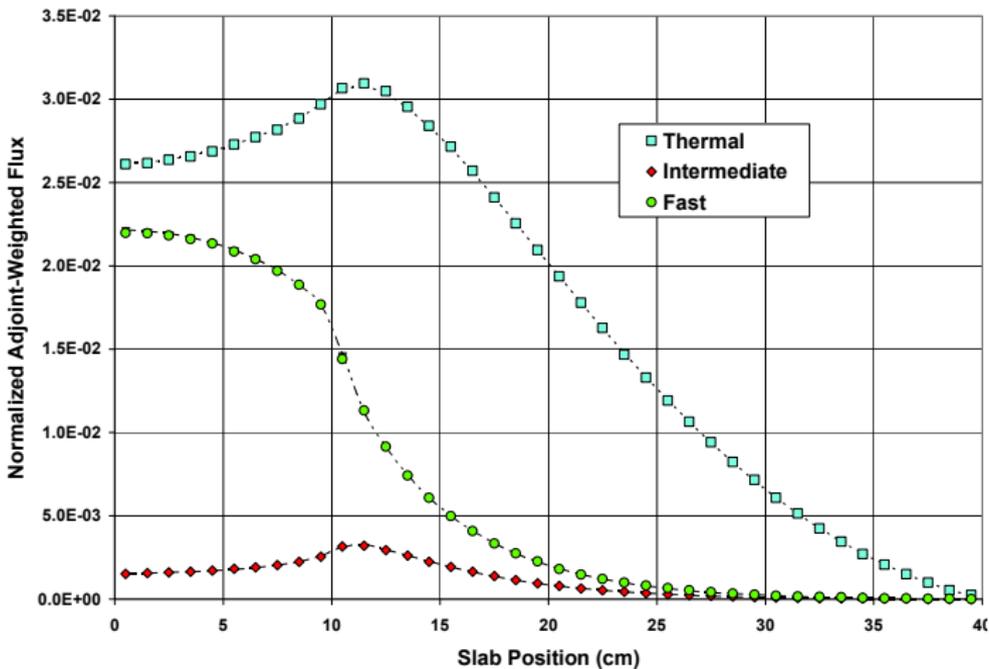
## Results: Adjoint-Weighted Flux

- Estimate adjoint-weighted flux averaged over some region  $R$ .

$$\overline{\psi^\dagger \psi} = \left\langle \psi^\dagger, \psi \right\rangle_R.$$

- Compare deterministic ( $S_n$  in Partisn) and Monte Carlo solutions.
  - Simple 3-group slab problem.
  - Two regions: left is fuel and right is moderator.
  - Reflecting boundary on left, vacuum on right.

## Results: Adjoint-Weighted Flux



# Outline

- 1 Introduction & Basics
- 2 Adjoint Weighting in Continuous-Energy Monte Carlo
- 3 Point Kinetics**
- 4 Sensitivity/Uncertainty
- 5 Graduate Student Research
- 6 Future Research

## Point Kinetics Model

- The point kinetics model is a simple description of neutron behavior in a reactor.
- Assumes flux is time separable and the spatial flux is distributed as the fundamental mode.

$$\frac{dn}{dt} = \left( \frac{\rho - \beta_{\text{eff}}}{\Lambda} \right) n(t) + \sum_i \lambda_i C_i(t) + q$$

$$\frac{dC_i}{dt} = \frac{\beta_i}{\Lambda} n(t) - \lambda_i C_i(t), \quad i = 1 \dots 6$$

- How are the parameters in this model estimated?

## Point Kinetics Parameters

- The point kinetics parameters may be measured or calculated.
- The parameters are ratios of adjoint-weighted integrals:

$$\Lambda = \frac{\langle \psi^\dagger, \frac{1}{v} \psi \rangle}{\langle \psi^\dagger, F \psi \rangle}$$

$$\beta_{\text{eff}} = \frac{\langle \psi^\dagger, B \psi \rangle}{\langle \psi^\dagger, F \psi \rangle}$$

- Here  $v$  is the neutron speed,  $F$  is the fission integral, and  $B$  is the fraction of the fission integral producing delayed neutrons.

## Point Kinetics Parameter Verification and Validation

- Show that the method is correctly estimating point kinetics parameters by:
  - Verification with analytic solutions.
  - Verification with another method (Partisn).
  - Validation with experimental delayed critical Rossi- $\alpha$  measurements.

$$\alpha_{DC} = -\frac{\beta_{\text{eff}}}{\Lambda}.$$

## Analytic Solution

- Two-group, infinite-medium problem with two delayed neutron groups:

$$\Lambda = \frac{\frac{1}{v_1} \frac{\Sigma_{s12}}{\Sigma_{R2}} + \frac{1}{v_2} \frac{\Sigma_{s12}}{\Sigma_{R2} - \xi_2 \nu \Sigma_f}}{\left\{ \frac{\Sigma_{s12}}{\Sigma_{R1}} [(1 - \beta) + \xi_1] + \xi_2 \right\} \frac{\nu \Sigma_f \Sigma_{s12}}{\Sigma_{R2} - \xi_2 \nu \Sigma_f}} = 44/3 \text{ ns},$$

$$\beta_{\text{eff}} = \frac{\frac{\Sigma_{s12}}{\Sigma_{R1}} \xi_1 + \xi_2}{\frac{\Sigma_{s12}}{\Sigma_{R1}} [(1 - \beta) + \xi_1] + \xi_2} = 1/2,$$

$$\alpha = - \frac{\left[ \frac{\Sigma_{s12}}{\Sigma_{R1}} \xi_1 + \xi_2 \right] \frac{\nu \Sigma_f \Sigma_{s12}}{\Sigma_{R2} - \xi_2 \nu \Sigma_f}}{\frac{1}{v_1} \frac{\Sigma_{s12}}{\Sigma_{R2}} + \frac{1}{v_2} \frac{\Sigma_{s12}}{\Sigma_{R2} - \xi_2 \nu \Sigma_f}} = -3/88 \text{ ns}^{-1}.$$

- $\xi_g$  is defined as the sum, over precursor index  $i$ , of all  $\chi_{ig} \beta_i$ .

## Analytic Solution Results

	Analytic	MCNP	C/E
$\Lambda$ (ns)	14.66667	$14.66548 \pm 0.00110$	0.99992
$\beta_{\text{eff}}$	0.50000	$0.50003 \pm 0.00005$	1.00006
$\alpha$ ( $\text{ns}^{-1}$ )	$-3.40909 \times 10^{-2}$	$-3.40955 \pm 0.00044 \times 10^{-2}$	1.00013

Comparison with Partisn ( $S_n$ )

#	G	Description
1	4	Bare fast slab
2	4	Metal slab with a moderating reflector
3	2	Metal slab, thermal absorber, and moderating reflector
4	8	Bare intermediate spectrum slab
5	4	Bare fast sphere
6	4	Reflected fast sphere
7	4	Subcritical bare fast slab ( $k = 0.78$ )
8	4	Supercritical bare fast slab ( $k = 1.14$ )

## Comparison with Partisn ( $S_n$ ) Results

#	Partisn	MCNP	C/E	$I$
1	9.79325 ns	9.79675 $\pm$ 0.00188 ns	1.00036	0.99021
2	135.19020 us	135.22164 $\pm$ 0.03384 $\mu$ s	1.00023	1.14537
3	49.16822 ns	49.20663 $\pm$ 0.01863 ns	1.00078	0.00488
4	112.05232 us	112.29905 $\pm$ 0.13692 $\mu$ s	1.00220	1.11580
5	1.72115 ns	1.72121 $\pm$ 0.00032 ns	1.00003	0.86498
6	10.18997 ns	10.18794 $\pm$ 0.00233 ns	0.99980	0.56477
7	10.17161 ns	10.17110 $\pm$ 0.00230 ns	0.99995	1.05365
8	9.67254 ns	9.67168 $\pm$ 0.00166 ns	0.99990	0.96534

## Comparison with Experimental Benchmarks

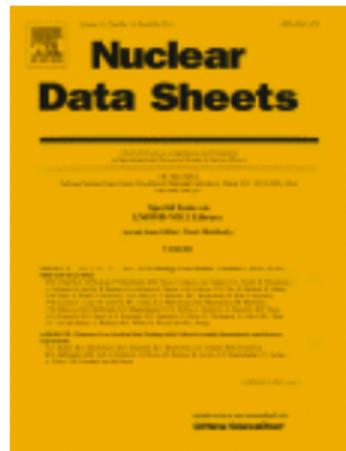
- Comparisons of Monte Carlo (MCNP) calculations with ENDF/B-VII.0 data and experimental measurements of Rossi- $\alpha$  (in  $\text{ms}^{-1}$ ).

	Experiment	MCNP	C/E
Godiva	$-1100 \pm 20$	$-1139.57 \pm 2.35$	1.017
Jezebel-239	$-640 \pm 10$	$-640.238 \pm 2.374$	1.000
BIG TEN	$-117 \pm 1$	$-115.518 \pm 0.219$	0.987
Jezebel-233	$-1000 \pm 10$	$-1071.18 \pm 3.50$	1.071
Flattop-233	$-271 \pm 3$	$-292.401 \pm 0.808$	1.079
Stacy-29	$-0.122 \pm 0.004$	$-0.122155 \pm 0.00296$	1.001
WINCO	$-1.1093 \pm 0.0003$	$-1.11723 \pm 0.00311$	1.007



## End-User Application: Nuclear Data Validation

- Prediction of experimental measurements inform the development of any nuclear data library.
- The latest US library, ENDF/B-VII.1, used this capability in MCNP to benchmark to  $\beta_{\text{eff}}$  and  $\alpha_{DC}$  measurements for the first time.
- Error found in the  $\beta_i$  of all actinides in previous library ENDF/B-VII.0; fixed in ENDF/B-VII.1.
- After fix, results show generally good agreement with measurement.



## Remarks

- Verification and validation demonstrates that the method (implemented in MCNP) can calculate point kinetics parameters accurately.
- Calculation requires minimal user interaction: one user parameter and one additional line of input.
- Only small cost in computational time and memory usage.
- Variant of this method being implemented in Serpent.
- Allows for additional benchmarking of kinetic measurements with current experiments.

# Outline

- 1 Introduction & Basics
- 2 Adjoint Weighting in Continuous-Energy Monte Carlo
- 3 Point Kinetics
- 4 Sensitivity/Uncertainty**
- 5 Graduate Student Research
- 6 Future Research

## Sensitivity Coefficient

- The sensitivity coefficient is the ratio of the relative change in a response  $R$  caused by some relative change in some system parameter  $x$ :

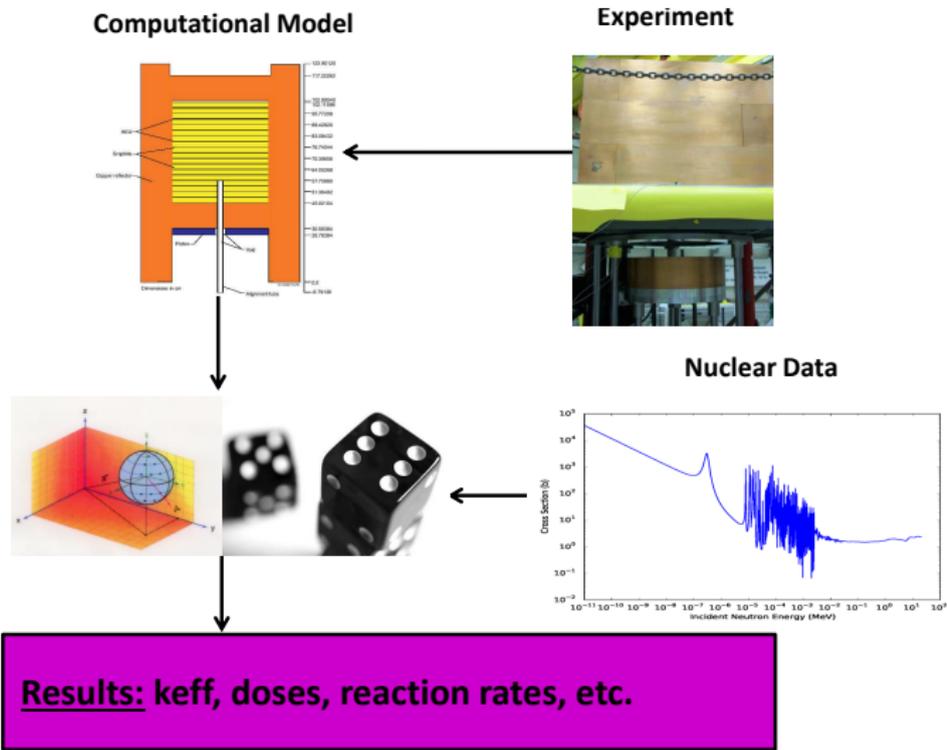
$$S_{R,x} = \frac{\Delta R/R}{\Delta x/x} = \frac{x}{R} \frac{\Delta R}{\Delta x}$$

- For this talk,  $R$  will be taken to be  $k_{\text{eff}}$  and  $x$  is some segment of nuclear data (e.g., a reaction cross section for a particular isotope in some energy range).

## Why Sensitivity Coefficients Matter in Criticality

- The sensitivity coefficient shows what nuclear data most impact  $k_{\text{eff}}$ .
- Empirically, most of the computational bias (inability of software to predict measurements) in  $k_{\text{eff}}$  is because of uncertainty in the nuclear data.
- Sensitivity coefficients offer a relatively simple way of showing if two systems are neutronically similar.
- The sensitivity coefficients can therefore be used to assess the predictive capability of software for a particular system based on how well it predicts neutronically similar experimental benchmarks.
- When convolved with cross section covariance data, can get estimates of uncertainties in  $k_{\text{eff}}$ .

# Schematic of a Calculation



## History of Sensitivity Analysis in Criticality

- Sensitivity analysis is a well established technique.
  - Has been used for decades with deterministic methods at Argonne National Laboratory, mostly for fast reactor uncertainty quantification.
  - Multigroup Monte Carlo sensitivity/uncertainty analysis has been in SCALE out of Oak Ridge National Laboratory for about a decade.
- Multigroup methods require two Monte Carlo calculations, one forward and one adjoint, and must consider the effect of multigroup collapse (not always easy).
- The adjoint weighting method for kinetics can be applied to sensitivity analysis as well, requiring minimal user input.
- This method has been implemented in MCNP and will be in the next version of SCALE.

## Sensitivity Coefficients with Adjoint Weighting

- Perturbation theory can be used to derive an expression for the sensitivity coefficient:

$$S_{k,x} = - \frac{\langle \psi^\dagger, (\Sigma_x - C_x - \frac{1}{k} F_x) \psi \rangle}{\langle \psi^\dagger, F \psi \rangle}.$$

- $\Sigma_x$  is the macroscopic cross section of interest (zero if not a cross section).
- $C_x$  is the scattering integral for nuclear data  $x$  (zero if not scattering).
- $F_x$  is the fission integral for nuclear data  $x$  (zero if not fission).

## Sensitivity Coefficients for Distributions

- Fission  $\chi$  and scattering distributions have an additional constraint that the area under the curve must be unity.
- The sensitivity coefficient may be adjusted accordingly:

$$\hat{S}_{k,f}(E', E, \mu) = S_{k,f}(E', E, \mu) - f(E' \rightarrow E, \mu) \iint dE d\mu S_{k,f}(E', E, \mu)$$

- The quantity  $\hat{S}_{k,f}$  is often referred to as the constrained or renormalized sensitivity coefficient.

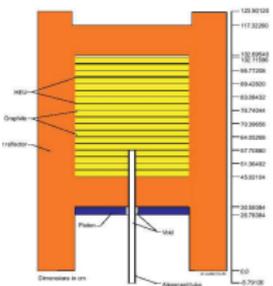
## Sensitivity Coefficients for Scattering Moments

- Often the scattering distributions and uncertainties are given as Legendre moments.
- Can express renormalized sensitivity coefficient  $\hat{S}_{k,f}(\mu)$  as Legendre moment sensitivity  $\hat{S}_{k,f,\ell}$ .
- Given a defined cosine grid with  $N$  bins with index  $i$ , the  $\ell$ th Legendre moment sensitivity is

$$\hat{S}_{k,f,\ell} = \frac{2\ell + 1}{2} f_\ell \sum_{i=0}^{N-1} (\mu_{i+1} - \mu_i) \frac{P_\ell(\mu_{i+1/2})}{F_{i+1/2}} \hat{S}_{k,f,i+1/2}$$

- $P_\ell(\mu_{i+1/2})$  is the  $\ell$ th Legendre polynomial at the cosine bin center.
- $f_\ell$  is the  $\ell$ th Legendre moment of  $f$ .
- $F_{i+1/2}$  is the cumulative density function of  $f$  integrated over the cosine bin.

# Sensitivity Result



## Sensitivity Coefficient Verification

- Show that the method is correctly estimating sensitivity coefficients by:
  - Verification with analytic solutions.
  - Verification with another method (multigroup Monte Carlo in SCALE/TSUNAMI).
  - Verification with direct perturbations.

## Sensitivity Coefficient Verification: Analytic Solution

- Infinite medium problem with three energy groups:

$g$	$\sigma_t$	$\sigma_c$	$\sigma_f$	$\nu$	$\chi$	$\sigma_{sg1}$	$\sigma_{sg2}$	$\sigma_{sg3}$
1	2	1/2	0	–	5/8	1	1/2	0
2	4	1	0	–	1/4	0	1	2
3	4	1/2	3/2	8/3	1/8	0	0	2

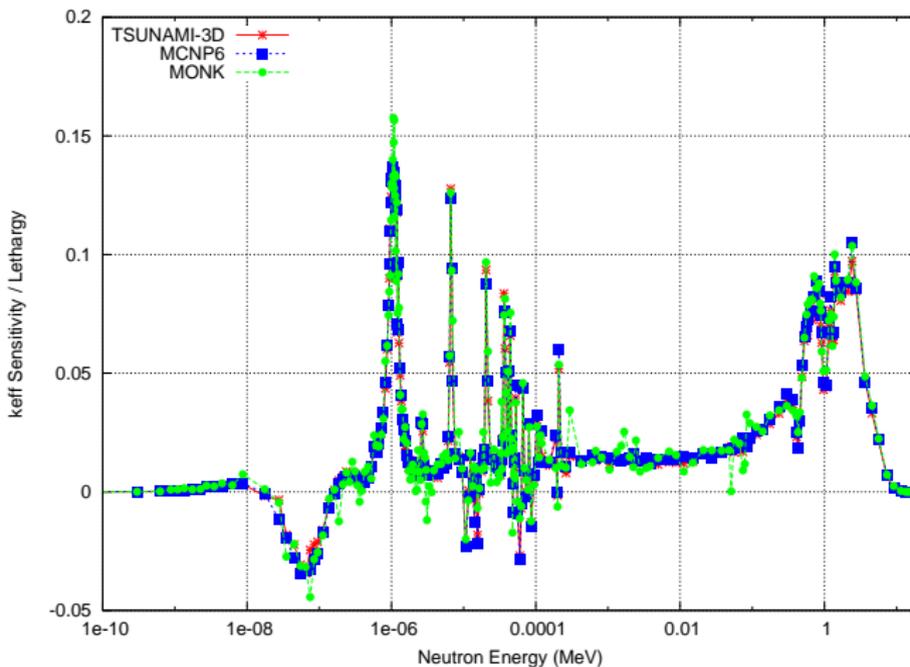
## Sensitivity Coefficient Verification: Analytic Solution

$x$	Exact $S_{k,x}$	MCNP6 $S_{k,x}$	$C/E$
$\sigma_{c1}$	$-5/24$	$-0.20868 \pm 0.10\%$	1.002
$\sigma_{c2}$	$-1/4$	$-0.24993 \pm 0.07\%$	0.999
$\sigma_{c3}$	$-1/4$	$-0.24985 \pm 0.05\%$	0.999
$\sigma_{f3}$	$+1/4$	$+0.25045 \pm 0.16\%$	1.002
$\nu_3$	$+1$	$+1.00000 \pm 0.00\%$	1.000
$\sigma_{s12}$	$+5/24$	$+0.20810 \pm 0.16\%$	0.999
$\sigma_{s23}$	$+1/4$	$+0.25083 \pm 0.15\%$	1.003

$g$	Exact $\hat{S}_{k,\chi_g}$	MCNP6 $\hat{S}_{k,\chi_g}$	$C/E$
1	$-5/24$	$-0.20805 \pm 0.12\%$	0.999
2	$+1/12$	$+0.08339 \pm 0.28\%$	1.001
3	$+1/8$	$+0.12465 \pm 0.17\%$	0.997

## Sensitivity Coefficient Verification: TSUNAMI Comparison

- 3-D MOX lattice in water. H-1 elastic cross section sensitivity:

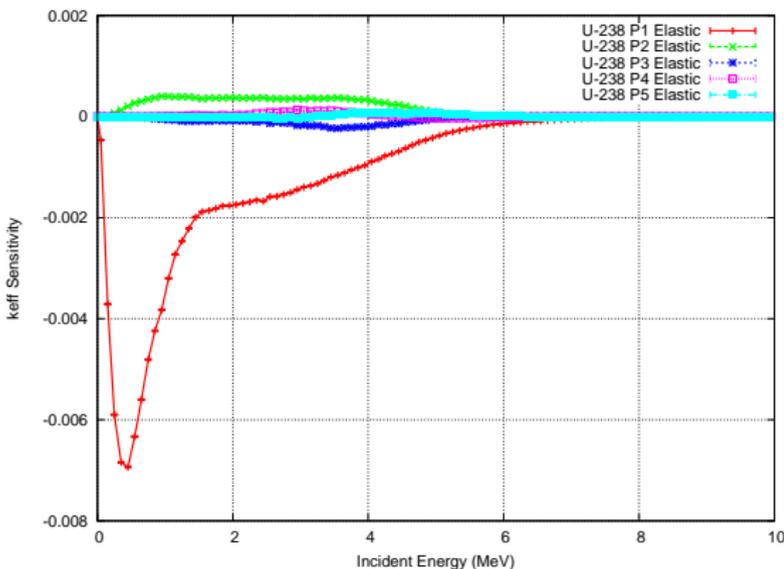


## Sensitivity Coefficient Verification: Direct Perturbation

- Spherical core, cylindrical (non-fissionable) reflector.
- Two energy groups, reflector has  $P_1$  anisotropy.
- Perturb reflector  $P_1$  scattering cross section by 10% find  $\Delta k$  and compare with that from adjoint method.
- Direct  $\Delta k = -0.00344(4)$ .
- Adjoint  $\Delta k = -0.00341(1)$ .
- Agrees within  $1-\sigma$ .

## Legendre Moment Sensitivity

- Flattop (HEU sphere with spherical natural uranium reflector).
- U-238 Elastic scattering sensitivities as function of incident neutron energy.



## Sensitivity Coefficient Applications

- Implemented in MCNP. Currently funded \$250k/year to support uncertainty quantification work.
- May be used in support of critical experiment design (required by the NNSA Nuclear Criticality Safety Program).
- Legendre moment sensitivities for scattering distribution has typically been neglected in nuclear data adjustments; it is now possible to include this.

## Sensitivity Coefficient Applications

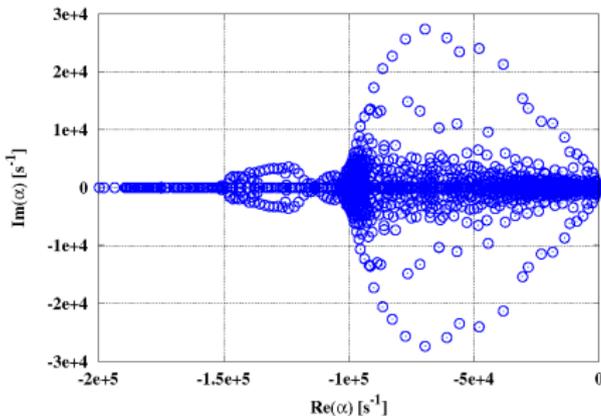
- Support criticality safety validation project for restart of plutonium production at Los Alamos National Laboratory:
  - Develop computational software that automatically finds neutronically similar benchmark experiments.
  - Uses information to compute computational bias and subcritical margins with cross section uncertainties.
  - Technically-based validation required for safe operations with fissionable material.

# Outline

- 1 Introduction & Basics
- 2 Adjoint Weighting in Continuous-Energy Monte Carlo
- 3 Point Kinetics
- 4 Sensitivity/Uncertainty
- 5 Graduate Student Research**
- 6 Future Research

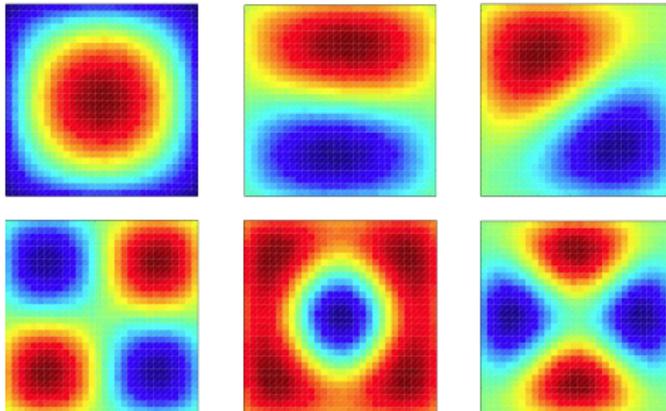
## $\alpha$ -Eigenvalue Spectra (Betzler, Michigan)

- Develop continuous-time adjoint Markov transition rate in forward calculation.
- Use linear algebra to solve for discrete approximations of forward and adjoint  $\alpha$ -eigenvalue spectra.
- Employ eigenfunction expansion to match time-dependent response measurements.
- Possible validation with new measurements on critical assemblies.



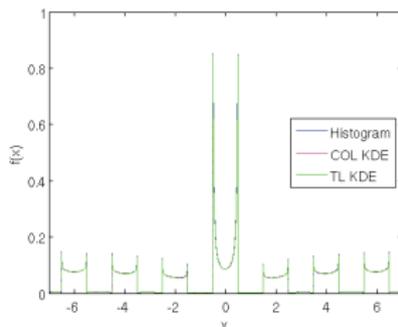
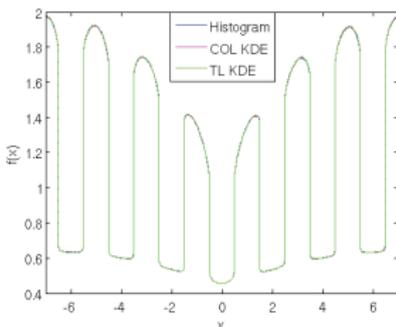
## Fission Matrix (Carney, Michigan)

- Use fission matrix to solve for  $k$ -eigenvalue modes.
- Applications:
  - Monte Carlo eigenvalue convergence acceleration and detection.
  - Correction of statistical uncertainties in eigenvalue calculations.
  - Higher-order perturbation theory and sensitivity analysis.



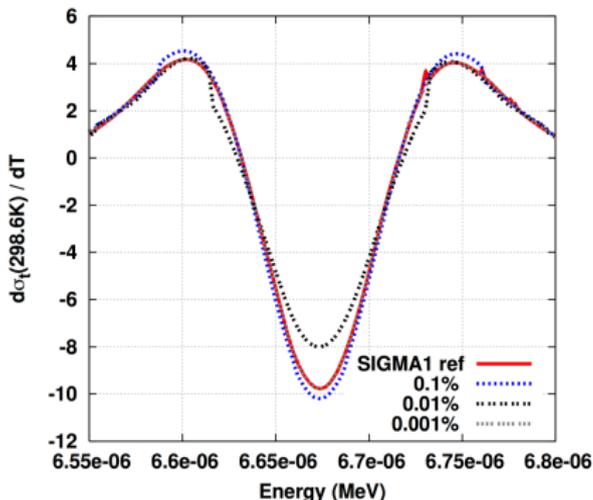
## Applied Kernel Density Estimators (KDEs) (Burke, Michigan)

- KDEs offer an alternative approach to tally responses at points.
- Extended to reaction rates in 1-D using mean-free-path based bandwidth.
- Implementation in OpenMC; current work on 2-D and 3-D.
- Possible future efforts:
  - Multiphysics with meshfree methods.
  - Depletion with node-based isotopics, temperatures.



## Doppler Reactivity Coefficients (Gonzales, UNM)

- Use sensitivity methodologies for Doppler reactivity coefficients.
- Use temperature series expansion by Yesilyurt to compute cross-section derivatives.
- Effective cross section shows reasonable agreement with direct perturbations.
- Current work is on scattering kernel derivative.



## Multi-point Kinetics (Clark, UNLV)

- Historically, there has been poor predictive capability of kinetics measurements of reflected metal systems.
- Spatially resolved, multi-point kinetics models exist that should give better agreement.
- Plan is to apply point kinetics methodologies in MCNP to implement multi-point kinetics in CE Monte Carlo.
- Since reflectors are often non-fissionable, need a collisional eigenvalue kinetics model.
- Goal is to predict and compare with kinetics measurements of a critical system with two coupled assemblies.



## CE Neutron Adjoint and FACEMC (Robinson, Wisconsin)

- Forward-Adjoint Continuous Energy Monte Carlo (FACEMC) software developed by Wisconsin.
- Software designed in a modular fashion to accommodate future development.
- Software is being developed with agile programming practices.
- Seeking permission to release as open source software to serve as a research development platform for radiation transport methods.



## Reactor Transients (Aures, GRS)

- New collaboration with GRS in Germany to investigate efficient approaches to Monte Carlo reactor transients with feedback.
- Possible research topics:
  - Application of higher- $\alpha$  modes.
  - Perturbation theory with  $\alpha$  eigenvalue.
  - Jacobian-Free Newton Krylov with Monte Carlo for neutronics solve.

# Outline

- 1 Introduction & Basics
- 2 Adjoint Weighting in Continuous-Energy Monte Carlo
- 3 Point Kinetics
- 4 Sensitivity/Uncertainty
- 5 Graduate Student Research
- 6 Future Research**

## Critical Experiment Design with Automated Optimization

- Critical experiments are often performed to assess a weakness in software predictive capability, which are driven by uncertainties in nuclear data.
- Sensitivity analysis provides a means to ensure that the critical experiment addresses that need.
- Sensitivity profiles can be combined with optimization methods to maximize value of the experiment (more information for less money).
- Possible approaches:
  - Derivative-based methods with higher-order perturbation theory.
  - Derivative-free methods with Mesh Adaptive Direct Search method.

## Sensitivity/Uncertainty of Transients

- Develop sensitivity methods for the fundamental  $\alpha$  eigenvalue.
- The time-dependent nature makes it sensitive to different nuclear data (downscattering, delayed emission fractions, etc.).
- More data from existing or new kinetic measurements of  $\alpha$ .
- Is it possible to reformulate eigenvalue equations like we are with the multi-point kinetics work to gain additional insights?

## Uncertainty Quantification with Temperature Dependence

- Nuclear data covariances are generated at discrete temperatures.
- Correlations exist between temperatures because of resonance parameter uncertainties.
- How would one handle correlations between temperatures?
- MC software Serpent can handle continuous temperature fields (uses Woodcock or delta tracking).
- How would one perform sensitivity/uncertainty analysis in the presence of continuous temperature fields.

## Time-Dependent KDE

- Transients and inertial confinement fusion (ICF) simulations need accurate time-dependent reaction or absorption/reemission rates.
- Discretization in time can lead to causality violations (radiation gets ahead of itself on the subsequent time step).
- Time-dependent KDEs at points with appropriate interpolation may offer a way to minimize this error.
- Possible approach is to have a causality-preserving boundary for KDE.

## Automated Variance Reduction

- In the last decade, significant progress has been made on automating variance reduction (VR) with weight windows and source biasing using deterministic methods.
- Optimized weight windows provide significant speedups for some problems (factors of 10-1000+).
- There are other VR techniques (exponential transforms, forced collisions, deterministic transport surfaces, etc.) available.
- Can deterministic methods be used to solve for optimal VR parameters of these other techniques as well?
- Can they be combined to make an even more efficient solution? Perhaps with adaptive learning?

## Summary

- My research has focused on developing applications to adjoint weighting in criticality simulations.
- These have been implemented in production Monte Carlo codes like MCNP, Serpent, SCALE, etc. and are robust and simple for an engineer to use.
- Point kinetics and sensitivity analysis have a close coupling with experimental measurements and the development of nuclear data.
- Graduate students who have worked with me have a diverse set of research interests in the area of criticality, reactor physics, and radiation transport simulation.