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Performance Assessment of Cost-Optimized Variance Reduction Parameters in Radiation Shielding Scenarios

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OUTLINE

Introduction & Background

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**Introduction & Background**

Objective: compare weight-dependent (WD) and -independent (WI) variance reduction (VR) parameter optimization approaches

- “Traditional” hybrid methods: minimize variance
  - AVATAR (Riper et al., 1997)
  - LIFT (Turner and Larsen, 1997)
  - FW-CADIS (Wagner and Haghighat, 1998)
  - State-of-the-art: FW-CADIS via ADVANTG (Mosher et al., 2015)
- This method: minimize computational cost (i.e., maximize FOM)
  - Computational cost inversely proportional to FOM
    \[
    \tilde{C} (\{VR\}) = \tilde{\sigma}^2 (\{VR\}) \tilde{\tau} (\{VR\})
    \] (1)
- Method implemented via COVRT (Solomon et al., 2014)
  - Calculate $\tilde{\sigma}^2$ and $\tilde{\tau}$ deterministically
  - Optimize these quantities by varying VR parameters
Methodology Overview

- Cost-optimized methods are Monte Carlo code agnostic
- Specific implementations are directly related to a Monte Carlo code
  - Descriptions of geometry, materials, tallies, sources, etc.
  - VR technique availability & implementation
  - Weight-dependent vs. weight-independent techniques
  - Performance measure for various physical & computational events
- Basic Steps
  1. Solve History-Score Moment Equations
  2. Solve Future Time Equations
  3. Calculate computational cost, $\tilde{C} (\{VR\}) = \tilde{\sigma}^2 (\{VR\}) \tilde{\tau} (\{VR\})$
  4. Vary VR parameters, perform 1–3, compare with previous cost
  5. Repeat, as necessary, until $\tilde{C} (\{VR\})$ is minimized
History-Score-Moment Equations (HSMEs) (1/4)

- Construct & discretize weight-augmented phase space:

\[ P = (x, \Omega, E, w) = (R, w) \]  

Define scoring functions and transport kernels
- How a particle contributes score in \( ds \) about \( s \) in its next event
  - Examples: \( p_\Sigma(P, s_\Sigma) \, ds_\Sigma, \, p_C(P, s_C) \, ds_C \)
- How a particle moves through phase space and changes weight
  - Analog emergence (single scattering) kernel example:

\[
E(P_0, P_1) \, dP_1 = \delta(x_1 - x_0) \, p(\Omega_0, E_0 \rightarrow \Omega_1, E_1) \times \delta(w_1 - w_0) \\
\times dx_1 \, d\Omega_1 \, dE_1 \, dw_1
\]  

- Assemble transport kernels into continuous random walks
History-Score-Moment Equations (HSMEs) (2/4)

- Analog collision with scattering example:

\[
\psi_E(P_0, s) \, ds = \int dP_1 \, T(P_0, P_1) \int dP_2 \Sigma(P_1, P_2) \\
\times \int ds_\Sigma \rho_\Sigma(P_2, s_\Sigma) \int dP_3 \, E(P_2, P_3) \psi(P_3, s - s_\Sigma) \, ds
\] (4)

- Assemble into history-score probability distribution function \( \psi(P_0, s) \, ds \)

\[
\psi(P_0, s) \, ds = \sum_i \psi_i(P_0, s) \, ds
\] (5)

- Probability of contributing score \( ds \) about \( s \) from phase space \( P_0 \)
History-Score-Moment Equations (HSMEs) (3/4)

- The $m$ moments of the history-score distribution are:

\[
M_m(P_0) = \int_{-\infty}^{\infty} s^m \psi(P_0, s) \, ds \tag{6}
\]

- First moment ($m = 1$) comparable to adjoint integral transport equation

- Associated detector responses (with physical source term $Q(P_0)$) are:

\[
\tilde{D}_m = \int M_m(P_0) Q(P_0) \, dP_0 \tag{7}
\]

- Population variance is:

\[
\tilde{\sigma}^2 = \tilde{D}_2 - \tilde{D}_1^2 \tag{8}
\]

- Difficulty of solving for $M_{m>1}(P_0)$ affected by VR techniques used
History-Score-Moment Equations (HSMEs) (4/4)

- Weight separability with weight-independent VR techniques (Booth and Cashwell, 1979):

\[ M_m (R, aw) = a^m M_m (R, w) = a^m w^m M_m (R, w = 1) \]  \hspace{1cm} (9)

- This separability shows that weight-independent techniques do not require a discretized weight mesh for any moments
  - Reduced memory requirements
  - Reduced deterministic solver computational time
  - Permits easier incorporation into pre-existing deterministic solver

Can WI techniques perform as well as WD techniques?
Future Time Equations (FTEs) (1/1)

- Construct future time distribution like history-score probability distribution:

\[
\Upsilon (P_0, \tau) d\tau = \sum_t \left[ \prod_{n_t=1}^{N_t} \int dP_{n_t} B_{n_t} (P_{n_t-1}, P_{n_t}) \tau_{n_t} (B_{n_t}, P_{n_t}) \right] d\tau \quad (10)
\]

- Each \( \tau_{n_t} \) from profiled Monte Carlo code calculations, \( \mathcal{O} \left( 10^{-7} \text{ minutes} \right) \)

- Similar to history-score-moment distribution, expected future time of particle at \( P_0 \) is:

\[
\overline{\tau} (P_0) = \int_0^\infty \tau \Upsilon (P_0, \tau) d\tau \quad (11)
\]

- Expected future time of a particle then calculated as:

\[
\tilde{\tau} = \int \overline{\tau} (P_0) Q (P_0) dP_0 \quad (12)
\]
**Tophat & Three-legged Duct, from Solomon (2010)**

![Diagram showing different regions with varying cross-sections and labels for surface tally, void, source, and cross-sections with values of $\Sigma_t$.

- Surface Tally
- Void
- $\Sigma_t = 1.0 \text{ cm}^{-1}$
- $\Sigma_t = 0.2 \text{ cm}^{-1}$
- $\Sigma_t = 2.0 \text{ cm}^{-1}$

---

**x [cm]**

**y [cm]**

---

**x [cm]**

**y [cm]**
Mini2Room, inspired by Kulesza et al. (2016)
**Tophat First and Second Moment Solutions**

![Diagram showing Tophat First and Second Moment Solutions](image-url)

- **$M_1$**
- **Optimized $M_2$**
Mini2Room First and Second Moment Solutions

$M_1$

Optimized $M_2$
**Tophat & Three-legged Duct Results**

- WI FOM ratio usually as good or better than WD
- WI $\sim 10 \times$ faster
- WI uses $\sim 100 \times$ less memory
- Means approach unity as the quadrature is refined
- Variance & FOM show no substantial change
- Relatively coarse quadrature can be effective
- Iterative optimization depends on relative changes

<table>
<thead>
<tr>
<th>Geometry</th>
<th>WD-WW$^a$</th>
<th>WI-I$^b$</th>
<th>WI-WW$^c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>TH$^d$</td>
<td>3.46</td>
<td>3.53</td>
<td>5.78</td>
</tr>
<tr>
<td>TLD$^e$</td>
<td>7.26</td>
<td>7.26</td>
<td>6.77</td>
</tr>
</tbody>
</table>

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$^a$ WD COVRT $\rightarrow$ Weight Windows  
$^b$ WI COVRT $\rightarrow$ Importances  
$^c$ WI COVRT $\rightarrow$ Importances $\rightarrow$ Weight Windows  
$^d$ Tophat  
$^e$ Three-legged Duct
Extended Tophat & Three-legged Duct FOM Ratios

![Graph showing extended Tophat and three-legged duct FOM ratios.](image-url)

- **Tophat, WD-WW**
- **3-Leg, WD-WW**
- **Tophat, WI-I**
- **3-Leg, WI-I**
- **Tophat, WI-WW**
- **3-Leg, WI-WW**

**Figure of Merit Ratio**

**Extension Length [cm]**
**Mini2Room Comparison with ADVANTG**

- ADVANTG ($S_4$, 27 neutron groups), FOM: 3.0
- COVRT ($S_4$, 2, 4, 8, and 27 neutron groups), FOMs: 15, 6.3, 3.8, 6.5
  - Varying FOM due to multi-group COVRT calculations collapsed to one-group importances
  - A variety of collapsing schemes were explored, but none gave consistent or consistently improved FOMs
- ADVANTG time: 8 seconds, MCNP time: 65 hours
  - Long-running histories in MCNP drove time up
- COVRT time: 0.3, 1.0, 5.3, 48.0 hours, MCNP time: 34–116 minutes
  - COVRT time scales linearly with workload
  - Multiple optimization passes for each energy group
Summary & Future Work

- Demonstrated the effectiveness of cost-optimized VR parameters
- Optimized WI techniques can be as, or more, effective than WD
  - Significant savings in deterministic runtime & memory requirements
- Varying levels of agreement between mean and variance values
- Optimization depends on relative changes
- Highly angle-dependent problems challenge these methods
- Value in creating hybrid radiation transport method benchmark suite
- Extension of history-score-moment and future-time equations to DXTRAN
Questions?

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Backup Slides
Mini2Room Optimized Second Moment, $S_4$ vs. $S_{12}$
**Tophat Second Moment & Importances**

- Optimized $M_2$
- Optimized Importances
Mini2Room Second Moment & Importances

Optimized $M_2$

Optimized Importances
References


References


