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Title: INSIGHTS INTO STRATIFIED SPLITTING TECHNIQUES FOR MONTE CARLO NEUTRON TRANSPORT (U)

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Insights into Stratified Splitting Techniques for Monte Carlo Neutron Transport (U)

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Abstract. Several different stratified splitting techniques for sampling the distance to collision are compared with the standard (unstratified) weight window splitting used in MCNP [1]. The results indicate that typical neutron penetration problems could be modestly more efficient using one of the stratified splitting techniques.

As a cautionary note, a very highly scattering neutron penetration problem shows that some stratification techniques do not always reduce the variance of the score distribution.

1 Introduction

This paper compares several possible ways to stratify the distance to collision sampling. The methods fall into two basic categories.

1. techniques that stratify the distance to collision sampling within a spatial cell, but not across spatial cells.
2. techniques that stratify the distance to collision sampling across spatial cells.

The paper will discuss the current MCNP[1] weight window technique, a simple cell based stratification technique, and a more sophisticated stratification technique described in [2,3]. The techniques will then be used for Monte Carlo calculations of the neutron penetration of a concrete slab. Analysis of the calculational results then drives development and testing of several modifications of the stratification techniques.

2 The MCNP Weight Window

The weight window in MCNP consists of a lower weight bound (w_{lj}) in each phase-space region j, a surviving weight level(w_{sj}), and an upper weight bound (w_{uj}). Two global multiplying constants, $1 \leq c_s \leq c_u$, specify the surviving weight and the upper weight bound as $w_{sj} = c_s w_{lj}$ and $w_{uj} = c_u w_{lj}$. If a particle's weight, w, is between the lower and upper weight bounds, then the particle is within the window and no action is taken. If the weight is below the window, then a roulette game is played and the weight is either set to zero (i.e. the particle is killed) or to w_{sj}. If the weight is above the window, the particle is split by the minimum integer m that puts the split weight (w/m) within the window; that is, the minimum m such that $w_{lj} \leq w/m \leq w_{uj}$. (MCNP requires $w_{uj} \geq 2w_{lj}$ so that there is always an integer split that will put the split weights within the window.)

After a weight window split, each of the m split particles is sampled independently. MCNP samples the distance to collision from

$$s(\xi) = -ln(1-\xi)/\sigma \tag{1}$$

where ξ is a uniform random number on (0,1) and σ is the total macroscopic cross section in the current cell. If $s(\xi)$ is greater than the distance to the surface of the current cell, then no collision occurs and the particle is put on the surface. The distance to collision is then resampled from the surface. That is, MCNP is "memoryless" because it forgets how many free paths the particle traveled before reaching the boundary.

Note that the m split particles might all collide very close to each other, or they might all reach the cell surface. To ameliorate this problem, the distance to collision sampling can stratified in a number of ways, some of which will be tested herein.

The phase-space region index j will be suppressed on w_{lj}, w_{sj}, and w_{uj} from now on as j always will be the particle's current phase-space region.

In this paper, the upper weight bound was always chosen 5 times larger ($c_u = 5$) than the lower weight bound. The survival weight for roulette varied with the survival weight sometimes being 1,2, or 3 times ($c_s = 1, 2, \, or \, 3$) larger the lower weight bound.

3 C Stratification

The stratification method described in this section is called C stratification because the stratification ends at the cell surface. That is, the stratification is done on a cell by cell basis.

The variance associated with the distance to collision samplings of the m particles can be reduced by stratification. Specifically, rather than use m independent random numbers, ξ_i to get m uncorrelated distances to collision

$$D_i = s(\xi_i) \qquad 1 \leq i \leq m \tag{2}$$

one can use one random number and get stratified distances

$$S_i = s((i-1+\xi)/m) \qquad 1 \leq i \leq m \tag{3}$$

One further modification is that if k of the particles reach the next surface without collision, then these k particles are combined into one particle of weight $k(w/m)$ at the surface.

4 S Stratification

Another stratification technique is described in [2]. The stratification is done using an importance function and the stratification is applied over the entire flight path of the particle. This technique was implemented in the SAMCE Monte Carlo code [3], [4], and [5]. The MCNP implementation will be called S stratification.

Following the notation of reference [2], let

1. $K(P' \rightarrow P)dP =$ the expected number of collisions (analog sampling) in dP about P due to a particle in free-flight at P'.
2. $\tilde{K}_I(P' \rightarrow P)dP = K(P' \rightarrow P)dP\ (I(P)/I(P'))$ the expected number of collisions (biased sampling) in dP about P due to a particle in free-flight at P'.

Note that the number of collisions at P has been increased from $K(P' \rightarrow P)dP$ to $\tilde{K}_I(P' \rightarrow P)dP$ so that to keep the procedure unbiased, the weight upon collision is multiplied by the factor

$$w_{mult}(P) = \frac{K(P' \rightarrow P)dP}{\tilde{K}_I(P' \rightarrow P)dP} = \frac{I(P')}{I(P)} \tag{4}$$

The j-th collision distance, s_j is sampled using a random number ξ by solving

$$(j-1) + \xi = \int_0^{s_j} \tilde{K}_I(P' \rightarrow P)dP \tag{5}$$

This process stops when the problem boundary is reached at a distance b. The number of collisions before the boundary is reached is

$$j_b = Integer\Big[\int_0^b \tilde{K}_I(P' \rightarrow P)dP - \xi + 1\Big] \tag{6}$$

Note that the number of collisions before the boundary is reached can vary at most by 1. In particular, Eq. 6 is monotonic so that with $\xi = 1$ and $\xi = 0$ one infers

$$Integer\Big[\int_0^b \tilde{K}_I(P' \rightarrow P)dP\Big] \leq j_b \leq Integer\Big[\int_0^b \tilde{K}_I(P' \rightarrow P)dP + 1\Big] \tag{7}$$

5 Using Weight Window Information for the S Stratification

The S method was implemented in a modified version of MCNP by setting a target weight, $w_t(P)$, that is within the MCNP weight window for each phase-space region. Because the weight windows are inversely proportional to the importance function estimated by MCNP, these target weights incorporate the importance function of Eq. 4. That is,

$$\tilde{K}_I(P' \to P)dP = K(P' \to P)dP\,\frac{w(P')}{w_t(P)} \tag{8}$$

where $w(P')$ is the weight of the particle at P'. The weight multiplication for sampling from $\tilde{K}_I(P' \to P)$ is

$$w_{mult}(P) = \frac{K(P' \to P)dP}{\tilde{K}_I(P' \to P)dP} = \frac{w_t(P)}{w(P')} \tag{9}$$

so that the weight colliding at P is

$$w_{col}(P) = w(P')\frac{w_t(P)}{w(P')} = w_t(P) \tag{10}$$

That is, the weight colliding at P is always the target weight. For MCNP, the most obvious choice for the target weight is some constant times the lower weight bound. MCNP already has such a constant as an input parameter to the weight window. That is, the target weight can be chosen as $w_t(P) = c_s w_l(P) = w_s(P)$ and then the target weight will always be within the window.

In the modified version of MCNP that implements the S stratification, the weight windows are not used in their standard fashion, they are used only to bias the transport as per Eq. 8.

6 Comments on MCNP Terms

The results of MCNP calculations will often be displayed from a "tally fluctuation chart". A sample chart is given in Table 1.

Table 1. Sample Tally Fluctuation Chart

nps	mean	error	vov	slope	fom
1000	5.1687E-06	0.1669	0.0547	0.0	34
2000	4.7718E-06	0.1204	0.0917	7.0	35
3000	4.7047E-06	0.1021	0.0543	4.6	33
4000	4.5861E-06	0.0878	0.0395	4.1	33
5000	4.4158E-06	0.0773	0.0322	4.8	35

1. nps = the number of particles started from the source
2. mean = sample mean

3. error = fractional error, one standard deviation of the sample mean divided by the sample mean
4. vov = variance of the variance = variance of the sample variance divided by the square of the sample variance
5. slope = an indicator of the high score tail behavior. The estimated number of finite score moments is slope-1. This paper will not focus on the slope, so the reader need not understand this information. (The information is given for completeness in deference to those readers who are familiar with MCNP's slope estimate.)
6. fom = $1/(error^2 \times Time)$ where "error" is as defined in item 3 and "Time" is the computer time (in minutes) expended for nps source particles. This is the "figure of merit" which measures the efficiency of an MCNP calculation.

For brevity, only the relevant entries in the tally fluctuation charts will be displayed in the problem results for the following sections.

7 Problem 1: Penetration of 180 cm Concrete Slab

The first test problem is a penetration problem that has characteristics similar to numerous shielding problems. There is a point isotropic 14 MeV source at one side of 180 cm concrete slab with a tally of the number of particles penetrating the slab at the other side. The problem has been divided into cells by planes every 10 cm. The MCNP weight window generator produced the energy-dependent weight window (see Table 13 Appendix A). All comparisons for problem 1 use this weight window information.

The S stratification is compared to standard MCNP and cell stratification in the following section. Variants of the S and C methods are then treated in subsequent sections.

8 Results with C and S Stratification

In this section, the S results are compared against standard MCNP weight window results and the cell stratified splitting weight window results. The results are compared for a fixed number of particles. The target weights in the cells were chosen to be the MCNP survival weight.

Table 2 compares the three methods. Note that the S method which has a global stratification has the smallest error, the cell stratification has the next smallest error, and the standard (unstratified) MCNP has the largest error. This is intuitively what one would expect, better stratification results in smaller errors. The problem is that the S method is slow because it tracks the particles all the way to the problem boundary on each distance to collision sampling.

Table 2. Comparison of Basic Stratification Methods

nps	mean	error	vov	slope	fom	stratification
75000	3.9908E-06	0.0271	0.0027	10.0	359	none, MCNP
75000	4.1082E-06	0.0243	0.0018	10.0	449	C $c_s = 3$
75000	4.0835E-06	0.0199	0.0023	6.7	139	S

The geometry in this problem is very simple; for more complex geometries the S method likely would be even slower compared to standard MCNP because the ratio of the time tracking to the time colliding increases in complex geometries. This speculation has not been tested.

In the following sections, modifications to increase the efficiency of the S stratification are tested.

9 S1 Stratification Modification 1

Note from Table 2 and the fom definition in section 6 that if the 75000 particles in the S method could be followed as fast as the 75000 particles in the standard MCNP method, the fom for the S method would be $(0.0271/0.0199)^2 = 1.85$ times higher than for the standard MCNP. This section discusses a way to decrease the time required by the S method.

Note that the weight crossing a surface in the S method is

$$w = w(P')e^{-\int_0^s \sigma(P(t))\ dt} \tag{11}$$

where s in Eq. 11 is the distance to the surface from the starting point P'. Suppose w becomes very small compared to the target weight in the next cell? It seems reasonable at this point to stop the stratification and play roulette. If the target weight ($w_t = c_s w_l$) is more than G times the surface weight, then a roulette game is played and the particle either survives with probability $w/(w_t/G)$ and weight w_t/G, or is terminated. If the particle survives the roulette, the collision density $\tilde{K}_I$ is increased by the survival probability and the stratification process continues.

The method suggested in reference [3] is very similar to the S1 method. The only difference is that the survival weight in [3] is w_t rather than the w_t/G of the S1 method. That is, the SAMCE method plays a harsher roulette game than the S1 method. The author speculates, based on previous experience with other roulette games, that the more gentle S1 roulette game likely will be at least as efficient (fom) as the SAMCE roulette in almost all cases, and modestly more efficient in many cases.

Table 2 gives results for the S1 method as a function of G for comparison against the results in Table 2. The modification substantially improves the fom for this problem. As expected, the S1 and SAMCE methods with $G =$

Table 3. S1 Stratification as a Function of G

nps	mean	error	vov	slope	fom	G	stratification
75000	3.9903E-06	0.0194	0.0019	10.0	241	1000	S1
75000	4.0949E-06	0.0191	0.0016	10.0	259	100	S1
75000	4.0206E-06	0.0194	0.0022	4.9	268	20	S1
75000	4.0634E-06	0.0199	0.0022	10.0	262	10	S1
75000	3.9887E-06	0.0200	0.0021	10.0	266	3	S1
75000	4.1520E-06	0.0204	0.0020	10.0	264	2	S1
75000	4.0283E-06	0.0200	0.0019	10.0	252	20	SAMCE

20 give similar results; the small fom difference is probably not statistically significant.

Comparing the S1 runs in Table 3 with S run in Table 2, note that error is similar with and without truncation. Thus the truncation is saving time with little degradation in the error.

One caution is that the importance function in this problem is monotonically increasing (weight windows monotonically decreasing). If the target weights in successive cells along the flight path were 1, .1, .01, .001, .01, .1, .001, and .00001, and if G is too small, the S1 stratification might stop before reaching the most important cell having target weight .00001. An example of this type of weight window behavior is a well logging tool with a near and far detector with a Monte Carlo tally that is a linear combination of the near and far detectors. It is quite possible that the importance function could increase as the particles move from the source vicinity to the vicinity of the near detector, then decrease for awhile between the near and far detectors, before again starting to increase in the vicinity of the far detector. Although this is a concern, note that if the importance function between the near and far detectors changes by a factor of F, then choosing G equal to F should alleviate the problem for many calculations.

10 S2 Stratification Modification 2

Although the S1 method is more efficient than the S method, the C method is still substantially more efficient than the S1 method. The S1 method provides better stratification than the cell stratification method, so why is the C method more efficient? An advantage the cell stratification has over the S and S1 stratifications is that the cell stratification perfectly preserves the total weight. The sum of the collided weight in the cell plus the weight reaching the surface is always the original weight.

Note that the basic S method introduces a fluctuation in the total weight colliding in a cell because $\int_{cell} \tilde{K}_I(P' \rightarrow P)dP$ does not have to be an integer. For example, if the aforementioned integral is 1.5, then half the time one collision will occur in the cell and half the time two collisions will occur in the cell. This introduces a factor of two fluctuation in the total weight colliding in the cell.

This fluctuation in total weight colliding often can be eliminated by choosing a target weight such that $\int_{cell} \tilde{K}_I(P' \rightarrow P)dP$ is an integer. Note that the target weight in a cell can be anywhere between the lower ($w_l(P)$) and upper weight window bounds ($w_u(P)$) and still have the collided particle weights within the window. That is, using Eq. 8 the number of collisions in the cell, N_c, could range from

$$\int_{cell} K(P' \rightarrow P)\frac{w(P')}{w_u(P)}dP \le N_c \le \int_{cell} K(P' \rightarrow P)\frac{w(P')}{w_l(P)}dP \tag{12}$$

If the rightmost integral in Eq. 12 is less than one, then no integer number of collisions is possible and the target weight is set to $w_l(P)$. Otherwise, in keeping with MCNP's traditional strategy, let M_c be the smallest integer value of N_c that satisfies Eq. 12. Then solve

$$\int_{cell} K(P' \rightarrow P)\frac{w(P')}{w_t(P)}dP = M_c \tag{13}$$

for the target weight which will produce exactly M_c collisions. That is,

$$w_t(P) = \frac{w(P')}{M_c}\int_{cell} K(P' \rightarrow P)dP \tag{14}$$

The S2 method was tested on the concrete slab problem first by itself and then in conjunction with the roulette game of the S1 method. Table 4 gives the results.

Table 4. S2 Stratification Results

nps	mean	error	vov	slope	fom	G	stratification
75000	3.9694E-06	0.0150	0.0010	10.0	138		S2
75000	4.0466E-06	0.0151	0.0013	5.6	241	10	S2 and S1

These results are similar to the results when no care was taken to ensure that there was no fluctuation in total weight colliding in a cell. Presumably for this problem at least, the fluctuation in total collided weight (when there is at least one collision) is not a major source of variance.

Note that there can still be a fluctuation in total collided weight when the expected number of collisions is less than 1. The S3 modification described next will address this fluctuation.

11 S3 Stratification Modification 3

The S2 stratification is still worse than the cell stratification.

The previous section supplied a method for removing the fluctuations in the total collided weight when the expected number of collisions was greater than or equal to one. In a penetration problem, the cells having a fractional number of collisions preferentially will be far from the start of the particle's flight. Penetrating particles will tend to have collisions far from the start of the particle's flight. Thus, the fluctuation in total colliding weight is preferentially located towards the tally region.

The fluctuation in total colliding weight can be eliminated by ending the stratification procedure after the first cell for which the expected number of collisions in the next cell is less than one; that is,

$$\int_{cell} K(P' \to P)\frac{w(P')}{w_l(P)}dP < 1 \tag{15}$$

Note if the current weight is already in the weight window at P', then the target weight will be equal to the current weight and then the sampling kernel of Eq. 8 becomes the analog kernel. In this case, the expected number of collisions is less than one and the analog kernel is sampled (using the same random number for the distance to collision sampling as at the start of the flight at P'.) The particle either reaches the surface or collides within the cell. In both cases, a new distance to collision random number is required.

If the weight falls below the lower weight bound, the particle plays roulette and is either terminated, or its weight is set equal to the lower weight bound. The distance to collision or boundary crossing then is done as per the previous paragraph because the surviving particle is now within the window.

Table 5 gives the results of the S3 stratification. The fom is now comparable to the cell stratification method of Table 2.

Table 5. S3 Stratification Results

nps	mean	error	vov	slope	fom	stratification
75000	4.0171E-06	0.0215	0.0014	10.0	446	S3

12 C1 Stratification

Note that the cell stratification (C) exactly conserved the total particle weight, the split particles either collided or they reached the cell surface. On the other hand, the total weight colliding in the cell and the weight reaching the cell surface are both fluctuating. This fluctuation can be removed.

An alternative cell stratification technique (C1) divides the sampling into uncollided and collided parts. The expected uncollided weight is then put on the cell surface and the sampling of the collided weight is stratified. If the total collided weight is below the lower weight bound, then one collided particle is sampled. Otherwise, the number of stratified collided particles is the minimum number such that the collided particle weights are within the window. Table 6 gives the C1 stratification results. (For convenience, the C stratification results from table 2 are repeated in 6.)

Table 6. Comparison of C1 and C Stratification Methods

nps	mean	error	vov	slope	fom	stratification
75000	4.0321E-06	0.0231	0.0017	10.0	474	C1 $c_s = 2$
75000	4.1267E-06	0.0234	0.0017	10.0	476	C1 $c_s = 3$
75000	4.1082E-06	0.0243	0.0018	10.0	449	C $c_s = 3$

Note that any improvement in the fom is marginal at best. In any case, the difference is hidden in the statistical noise. Because method C has a fluctuation in the weight reaching the cell surface and method C1 does not, it is easy to design a problem where method C1 is definitely better than method C; this will be done in the next section.

13 Problem 2: Uncollided Concrete Slab Problem

As mentioned in the previous section, method C has a fluctuation in the weight reaching the cell surface and method C1 does not. If all the collided particles immediately terminate, then there is no collided contribution to the tally and the fluctuation in particle weight reaching the surface in method C will not be hidden in the statistical noise of the collision sampling. Thus, a problem is contrived from the previous concrete slab problem by choosing an energy cutoff of 13.999999999 (just below the 14 MeV source).

Furthermore, if the source is monodirectional and perpendicular to the slab, then the analytic solution is $exp(-\sigma T)$, where σ is the cross section at 14 MeV and $T = 180$ cm is the thickness.

14 Methods S, C, and C1 on the Uncollided Concrete Slab Problem

This section shows that the C1 method is indeed better than the C method when the statistical fluctuation from the collided contribution is eliminated. Note that for this problem, the S method is a zero variance method. A new weight window was generated for this problem (see Table 14 Appendix A). Table 7 compares the methods for this uncollided problem. As expected, the S method has zero variance, the C1 method is better than the C method, and standard MCNP is the worst of all.

Table 7. Stratification Methods for the Uncollided Problem

nps	mean	error	vov	slope	fom	stratification
250000	2.7969E-09	0.0171	0.0007	10.0	3099	none, MCNP
250000	2.7951E-09	0.0054	0.0005	10.0	33405	C $c_s = 3$
250000	2.7881E-09	0.0028	0.0000	10.0	96842	C1 $c_s = 3$
250000	2.7876E-09	0.0000	0.0000	10.0	∞	S

15 C2 Stratification

The C1 method is not a zero variance method because particles within the weight window are sampled normally, that is, without separating the particle into collided and uncollided parts. As a test, the C1 method was modified (method C2) so that if the weight was above the lower weight bound, the particle was then separated into collided and uncollided parts. For this problem, the C2 method always separates the particle into collided and uncollided parts because the particle weight is always above the lower window bound in this problem. (Note that the window roulette game is never played after collisions because the neutrons are terminated by energy cutoff.) Results for the C2 method are given in Table 8.

Table 8. The C2 Method for the Uncollided Problem

nps	mean	error	vov	slope	fom	stratification
250000	2.7876E-09	0.0000	0.0000	10.0	∞	C2

16 Carbon Slab Problem

In the concrete slab problems, the S, S1, and S2 methods all have lower errors at 75000 particles than the standard MCNP, C, and C1 methods. This is not always the case. For highly scattering problems, errors in the S method can be larger than the errors in standard MCNP calculations for a fixed number of particles. That is, in addition to losing on computer time, the sample variance can be higher for the S method than for the standard MCNP method.

The S, S1, and S2 methods all introduce a fluctuation in the total collided weight. The C and C1 methods only have this fluctuation in the last cell before the particles escape. Stated another way, if the problem were infinitely thick then the total weight colliding for the standard MCNP, C, and C1 methods is equal to the initial weight at the source or collision exit point P', whereas for the the S, S1, and S2 methods the total weight colliding varies. The effect of this fluctuation can be demonstrated in a problem with high scattering.

A 500 cm carbon slab of density 2.03 g/cm^3 is chosen for a highly scattering case. There is a point isotropic 1 eV source at one side of the carbon slab and a current tally at the other side. The slab is divided by planes parallel to the slab faces into 50 spatial cells, each 10 cm thick. MCNP's weight window generator produced the weight window of Table 15 used in the calculations shown in Table 9. Table 9 compares the standard MCNP, S, and S3 methods for 32000 particles.

Table 9. Comparison of Stratification Methods for Carbon Slab

nps	mean	error	vov	slope	fom	stratification
32000	1.4602E-06	0.0417	0.0062	10.0	7.5E+00	none, MCNP $c_s = 4$
32000	1.4869E-06	0.1120	0.0392	0.0	3.2E-01	S
32000	1.3539E-06	0.0316	0.0025	10.0	4.7E+00	S3

Note that the S method is very inefficient compared to standard MCNP. Even for the same number of particles, the error for the S method is much higher than the error for standard MCNP. On the other hand, the S3 method, designed to eliminate the fluctuation in total colliding weight, does have a smaller error and vov than standard MCNP at 32000 particles. The only reason that standard MCNP has a better fom is that standard MCNP runs 4.6 times as many particles per minute as S3. Thus, the S3 method is indeed doing as designed.

17 H Stratification

The H stratification method described on this section belongs to category 2 in section 1. The H method is designed to fit in with MCNP's structure that so that one need not keep track of the importance-weighted optical path as in Eq. 5. In one sense the method is a hybrid of the S and C methods. Under typical conditions, the H method can make sure the collisions are appropriately spaced apart, even across cell boundaries. On the other hand, the stratification can be forgotten in those cells where the MCNP user wishes to use other variance reduction techniques that make use of MCNP's traditional "memoryless" sampling from a cell boundary.

For the moment, assume that there are no other MCNP variance reduction techniques in use that need MCNP's traditional "memoryless" sampling from a cell boundary; this case will be treated later. A particle can be in the process of "entering a cell" in three ways:

1. The particle enters the cell by crossing a surface from the previous cell.
2. The particle enters the cell from the source.
3. The particle enters the cell upon exiting a collision. Note that although a particle may already have had multiple collisions in a cell, it is still treated as "entering the cell" at every collision exit.

At every source or collision exit point P', generate a random number ξ. This random number can be used to sample all collisions along the particle's current flight path in the following process.

1. When the particle enters a cell, first ensure that its weight is above the lower weight window bound. That is, play roulette if the particle is below the window and either kill the particle or increase its weight to the survival weight. Second, unless the entering particle is killed by roulette, separate the particle into the weight colliding before the cell boundary, w_c, and the weight arriving at the cell boundary, w_b. (If $w_c = 0$ then no collisions occur and all the weight enters the next cell, in which case go back to step 1 for the new cell.)
2. When $w_c \geq w_l$, choose the number of collisions to sample in the cell to be the smallest integer $M_c \geq 1$ such that the weight of the collided particles, w_c/M_c, satisfies
$$w_l \leq w_c/M_c \leq w_u \tag{16}$$
That is, the collided particles will have weights within the weight window. When $w_c < w_l$, choose $M_c = 1$ and rely on the weight window to roulette the particle upon exiting the collision, if the particle is below the window at that time.
3. Sample M_c stratified distances to collision, C_i, from the collided weight pdf. That is, sample C_i, according to
$$C_i = s_c((i-1+\xi)/M_c) \qquad 1 \leq i \leq M_c \tag{17}$$
where $s_c(\eta)$ is the formula for obtaining the distance to collision from the collided weight pdf using the random number η. Process the collided particles to their exit points P_i'.
4. For the particle reaching the cell boundary, the process repeats (in the next cell) from step 1 using the same random number ξ.

When considering interactions with other MCNP variance reduction techniques, note that that although the distances to collision can be correlated in every cell along the flight path using the same random number, the distances to collision need not be correlated in every cell. For example, if the flight path went through cells 1 to 5, it would be perfectly okay to use the procedure above with random number ξ in cells 1, 2, 3, and 5, but in cell 4 use MCNP's traditional distance to collision sampling with a different random number ξ'. If the particle crosses cell 4 without collision, then the collision sites in cell 5 can be stratified using the random number ξ, even though ξ was not used in cell 4.

Table 10 compares the C1 stratification of Table 6 against the H stratification.

Table 10. H Stratification

nps	mean	error	vov	slope	fom	stratification
75000	4.0351E-06	0.0165	0.0011	8.5	323	H $c_s = 1$
75000	3.9908E-06	0.0271	0.0027	10.0	359	MCNP, standard
75000	4.0321E-06	0.0231	0.0017	10.0	474	C1 $c_s = 2$

Note that the error for method H at 75000 particles is very good. The only previous method that beats this error is the combination of S2 and S1 methods in Table 5. On the other hand, the H method loses on fom relative to the C1 method because the number of source particles per minute in the methods are 6.5818E+03 and 1.7968E+04 respectively.

18 H1 Stratification

In H stratification, the particle is always split into its collided and uncollided weights. When the particle weight entering the cell is at the lower window bound, then the collided weight upon entering collision will be below the lower window bound and the collided weight exiting collision will also typically be below the lower window bound at the exiting energy. Additionally, when the particle weight entering the cell is at the lower window bound, then the uncollided particle typically has weight below the lower bound of the new cell being entered. Thus particles are constantly churning; they are produced by separation into collided and uncollided particles and terminated when they lose the weight window roulette game.

The H1 stratification method does not split the particle into collided and uncollided weights when the particle's weight is below the upper window bound. Instead, the particle's next collision or cell surface crossing is sampled from the analog kernel using the existing distance to collision random number. Upon collision or surface crossing selection from the analog kernel, the correlation stops and a new distance to collision random number is used. The H1 method is similar to the S3 method in this respect.

Comparing Table 11 with Table 10, one sees that H1 stratification is more efficient than H stratification. The error is higher when the stratification is stopped, but the reduction in computer time overcomes the higher error. Notice that H1 and C1 are very similar. This is not surprising as neither starts a stratification process when the weight is not above the window. The cells in the calculation are several free paths thick, so typically the uncollided weight reaching the next cell boundary in the penetration direction will not be above the weight window.

Table 11. H1 Stratification

nps	mean	error	vov	slope	fom	stratification
75000	3.9926E-06	0.0226	0.0015	10.0	450	H1 $c_s = 1$
75000	3.9867E-06	0.0239	0.0016	10.0	448	H1* $c_s = 2$
75000	4.0321E-06	0.0231	0.0017	10.0	451	C1 $c_s = 2$

19 Comments on Cell Size

The cells in the problems presented herein were chosen so that the spatially adjacent weight windows typically decreased by a factor of 2 to 5 in the penetration direction. The average mean free path in problem 1 was typically about 2.5 cm and the widths of the cells were 10 cm each. Under these circumstances, the S method usually has most of its stratified collisions in the cell where the particle exited collision. Thus the stratification provided by the S method is not much better than the C method which only stratifies within the cell.

If the cells are optically thin, the error with the C method should be similar to the standard MCNP because the most of the stratified particles reach the next boundary and then lose memory of the location of the last collision in the previous cell. The S stratification should be unaffected by the cell thickness (except for computer time) because the optical path does not depend on the number of cells in the geometry.

When the cells (of problem 1) are made 1 cm thick, instead of 10 cm thick, the thickness is then a fraction of a free path. The same weight window was used for 1 cm and 10 cm cells. Each of the 10 cm cells was split into ten 1 cm cells and all the 1 cm cells used the window of their original 10 cm cell. That is, cells 1 to 10, cells 11 to 20, etc. in the 1 cm case used the same weight windows as cell 1, cell 2, etc. in the 10 cm case.

Table 12 shows some of the same comparisons as Tables 2 and 11, but for the 1 cm cells rather than the 10 cm cells. Table 12 shows that the C method error is now comparable to the standard MCNP method rather than substantially better as in the 10 cm thick cell case. The H1 and S methods errors are statistically unaffected by the cell division.

Table 12. Problem 1 with Thin Cells

nps	mean	error	vov	slope	fom	stratification
75000	4.0727E-06	0.0264	0.0020	10.0	262	none, MCNP
75000	4.0131E-06	0.0268	0.0027	10.0	250	C $c_s = 3$
75000	3.9215E-06	0.0227	0.0016	10.0	267	H1 $c_s = 1$
75000	4.1064E-06	0.0244	0.0016	10.0	277	H1 $c_s = 3$
75000	4.0355E-06	0.0190	0.0016	10.0	29	S

20 Summary

This paper has analyzed and tested several possible techniques for stratifying the distance to collision sampling in neutron penetration problems.

Although the S stratification techniques usually reduced the score variance, the S stratification techniques often increased the time per history enough so that the resulting calculational efficiency was worse than the standard (unstratified) weight window splitting in MCNP. The S stratification techniques do well when there is very little scattering. When the scattering is very high however, the S techniques not only can increase the time per history, they can increase the score variance as well. An additional disadvantage of the S methods for Monte Carlo codes (such as MCNP) that assume that particles lose their memory at cell boundaries, is that the S methods may not be usable with other variance reduction techniques in the codes.

The C stratification techniques always reduced the score variance without increasing the time per history. Except for extreme cases such as the uncollided concrete slab problem, the simplest of the C stratification techniques did as well as the more complicated ones. For 14 Mev neutron penetration of 180 cm of concrete, the basic C method increased the calculational efficiency by a modest, but not insignificant, factor of 1.29.

An additional advantage of the C methods for Monte Carlo codes (such as MCNP) that assume that particles lose their memory at cell boundaries, is that the C methods can be used with other variance reduction techniques in the codes. That is, because the stratification stops at the cell boundary, the C methods preserve the memoryless structure that other variance reduction may require.

The hybrid method can be used with all MCNP variance reduction techniques also, by not using the same distance to collision random number in those cells using other variance reduction techniques. The hybrid coding is modestly more complex with little apparent benefit over the simpler C methods for the typical neutron shielding problems studied here. The efficiency gain due to better stratification is lost in the statistical noise when other sources of variance overwhelm the variance associated with the distance to collision sampling. On the other hand, the H1 method does provide a better stratification for the extra coding complexity and the efficiency is not apparently worse for typical problems.

From a purely theoretical point of view, the author prefers the H1 method, but code users might prefer the basic C method because the coding is modestly easier to read and understand.

References

1. X-5 Monte Carlo Team, "MCNP-A General Monte Carlo N-Particle Transport Code, Version 5," Los Alamos National Laboratory Report LA-UR-03-1987, April 24, 2003
2. M.H. Kalos and P. A. Whitlock, "*Monte Carlo Methods, Volume 1; Basics,*" John Wiley and Sons, Inc (1986). (see page 154)
3. Henry Lichtenstein, Martin O. Cohen, Herbert A. Steinberg, Eugene S. Troubetzkoy, Mendel Beer, "The SAM-CE Monte Carlo System for Radiation Transport and Criticality Calculations in Complex Configurations (Revision 7.0) Computer Code Manual CCM-8, Research Project 972, July 1979
4. M. O. Cohen, W. Guber, E. Troubetzkoy, H. Lichtenstein, H. Steinberg and M. Beer, "SAM-CE: A Three-Dimensional Monte Carlo Code for the Solution of the Forward Neutron and Forward and Adjoint Gamma-Ray Transport Equations Revision B," DNA-2830F-B (August 1973).
5. M. O. Cohen, et al., "SAM-CE: A Three-Dimensional Monte Carlo Code for the Solution of the Forward Neutron and Forward and Adjoint Gamma-Ray Transport Equations," MR-7021, DNA-2930F (November 1971).

21 Appendix - Weight Windows Used in the Monte Carlo Calculations

Table 13. Lower Weight Window Bounds for Concrete Slab Problem 1

E_{min}	0.000E-00	1.000E-02	3.000E-02	1.000E-01	3.000E-01
E_{max}	1.000E-02	3.000E-02	1.000E-01	3.000E-01	1.000E+00
cell	window 1	window 2	window 3	window 4	window 5
1	1.000E+02	1.000E+02	1.000E+02	1.338E+02	1.959E+02
2	4.421E+01	5.000E+01	5.981E+01	5.324E+01	3.100E+01
3	1.823E+01	1.296E+01	1.004E+01	1.177E+01	8.291E+00
4	5.455E+00	3.595E+00	6.075E+00	2.839E+00	2.426E+00
5	1.973E+00	8.522E-01	9.490E-01	6.518E-01	8.130E-01
6	7.115E-01	3.408E-01	2.979E-01	3.270E-01	2.409E-01
7	2.598E-01	1.361E-01	1.264E-01	1.239E-01	1.078E-01
8	9.600E-02	6.984E-02	6.191E-02	5.050E-02	4.544E-02
9	3.673E-02	2.599E-02	1.945E-02	1.533E-02	1.448E-02
10	1.370E-02	9.368E-03	6.799E-03	5.910E-03	4.543E-03
11	4.946E-03	2.348E-03	2.122E-03	2.242E-03	1.642E-03
12	1.807E-03	8.342E-04	7.686E-04	7.144E-04	5.646E-04
13	6.502E-04	2.969E-04	2.759E-04	2.969E-04	2.255E-04
14	2.477E-04	1.440E-04	1.179E-04	1.169E-04	8.690E-05
15	9.325E-05	5.005E-05	4.499E-05	4.268E-05	3.289E-05
16	3.492E-05	1.977E-05	1.887E-05	1.793E-05	1.487E-05
17	1.328E-05	8.033E-06	7.356E-06	7.628E-06	6.839E-06
18	5.475E-06	3.827E-06	3.758E-06	3.640E-06	3.608E-06

E_{min}	1.000E+00	2.000E+00	4.000E+00	8.000E+00
E_{max}	2.000E+00	4.000E+00	8.000E+00	1.000E+36
cell	window 6	window 7	window 8	window 9
1	6.997E+01	2.784E+00	5.664E-01	5.000E-01
2	7.190E+00	8.319E-01	2.218E-01	7.897E-02
3	2.671E+00	3.971E-01	9.518E-02	3.319E-02
4	1.371E+00	1.569E-01	4.340E-02	1.592E-02
5	3.235E-01	7.274E-02	1.969E-02	7.832E-03
6	1.354E-01	2.673E-02	7.989E-03	3.657E-03
7	5.785E-02	9.767E-03	3.516E-03	1.803E-03
8	2.396E-02	4.161E-03	1.785E-03	8.775E-04
9	6.138E-03	1.790E-03	7.783E-04	4.347E-04
10	2.125E-03	7.796E-04	3.650E-04	2.280E-04
11	8.529E-04	3.299E-04	1.708E-04	1.267E-04
12	3.501E-04	1.530E-04	8.865E-05	6.874E-05
13	1.575E-04	6.720E-05	4.576E-05	4.072E-05
14	5.856E-05	3.059E-05	2.186E-05	2.090E-05
15	2.479E-05	1.538E-05	1.137E-05	1.306E-05
16	1.232E-05	8.529E-06	6.844E-06	8.123E-06
17	5.984E-06	4.747E-06	4.415E-06	4.819E-06
18	3.333E-06	3.127E-06	3.061E-06	3.258E-06

Table 14. Lower Weight Window Bounds for Concrete Slab Problem 2

cell	window
1	5.000E-01
2	1.676E-01
3	5.572E-02
4	1.859E-02
5	6.282E-03
6	2.120E-03
7	7.104E-04
8	2.396E-04
9	7.961E-05
10	2.672E-05
11	8.997E-06
12	3.013E-06
13	1.013E-06
14	3.384E-07
15	1.136E-07
16	3.797E-08
17	1.268E-08
18	4.245E-09

Table 15. Lower Weight Window Bounds for Carbon Slab Problem

cell	window
1	2.500E-01
2	4.849E-02
3	2.944E-02
4	1.900E-02
5	1.320E-02
6	9.919E-03
7	8.055E-03
8	6.267E-03
9	5.032E-03
10	4.087E-03
11	3.189E-03
12	2.545E-03
13	2.065E-03
14	1.685E-03
15	1.366E-03
16	1.099E-03
17	8.950E-04
18	6.983E-04
19	5.737E-04
20	4.524E-04
21	3.772E-04
22	3.067E-04
23	2.373E-04
24	1.954E-04
25	1.497E-04
26	1.132E-04
27	8.856E-05
28	7.303E-05
29	5.690E-05
30	5.106E-05
31	4.169E-05
32	3.280E-05
33	2.696E-05
34	2.270E-05
35	1.848E-05
36	1.538E-05
37	1.250E-05
38	1.031E-05
39	8.398E-06
40	7.198E-06
41	5.939E-06
42	4.604E-06
43	3.688E-06
44	2.929E-06
45	2.318E-06
46	1.758E-06
47	1.522E-06
48	1.242E-06
49	1.097E-06
50	9.336E-07